Lesson: Derivative Techniques 3

❖ Obj – Finding Derivatives of composite functions using the Chain Rule

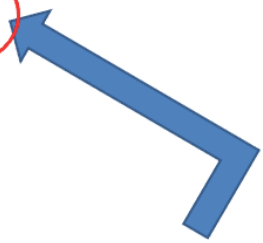
❖ Find $\frac{d}{dx}\left[(x)^{100}\right] = 100x^{99}$

$$\frac{d}{dx}\left[(x+1)^{100}\right]$$

❖ Find $\frac{d}{dx}\left[(x^2+1)^{100}\right]$

$$= 100 \cdot (x^2+1)^{99} \cdot 2x$$

$$= \boxed{200x \cdot (x^2+1)^{99}}$$



➢ O.K. but where did the 2x come from?

Examine:

$$\frac{d}{dx}[f(g(x))] = (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

or

$$\frac{d}{dx} f(stuff) = f'(stuff) \cdot (stuff)'$$

❖ <u>When all else fails, use M&Ms</u>

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'(g(x)) \cdot g'(x)$$

i. Differentiate the outer shell, leave the chocolate inside alone.

ii. Differentiate the chocolate inside.

<u>EX. 1</u>: Find dy/dx if $y = Cos(x^3)$

EX. 2: Find dy/dx, if $y = \tan(4x^3 + x)$

❖ But what about a weird one like:

Ex. 3: Find $f'(x)$ if

$$f(x) = \cos^4(7x^2)$$

$$\frac{d}{dx}\left[f\left(g(h(x))\right)\right] = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

i. Differentiate the shell, leave the chocolate & peanut alone.

ii. Differentiate the chocolate, but leave the peanut alone

iii. Differentiate the peanut

$$f(x) = \cos^4(7x^2)$$

$$f'(x) = 4 \cdot \cos^3(7x^2) \cdot (-\sin(7x^2)) \cdot (14x)$$

$$= \boxed{-56x \cdot \cos^3(7x^2) \cdot \sin(7x^2)}$$

EX.4 : Find $f'(x)$, *for* $f(x) = \tan^2 x$

EX.5: Find $\frac{d}{dx}\left[\sqrt{x^2+1}\right]$

EX. 6: Find

(a) $\frac{d}{dx}[\sin(2x)]$

(b) $\frac{d}{dx}[\tan(x^2+1)]$

(c) $\frac{d}{dx}[\sqrt{x^3 + \csc x}]$

(d) $\frac{d}{dx}[(1 + x^5 \cdot \cot x)^{-8}]$

(e) $\frac{d}{dx}\left[\frac{1}{x^3+2x-3}\right]$

EX. 7: Find

$\frac{d}{dx}\left[\sin\left(\sqrt{(1+\cos x)}\right)\right]$

EX. 8: Given the following table of values, find the indicated derivatives in parts (a) & (b).

x	f(x)	f'(x)	g(x)	g'(x)
4	-3	7	-3	-5
-3	-1	2	9	8

(a) $p'(4)$, where $p(x) = f(g(x))$

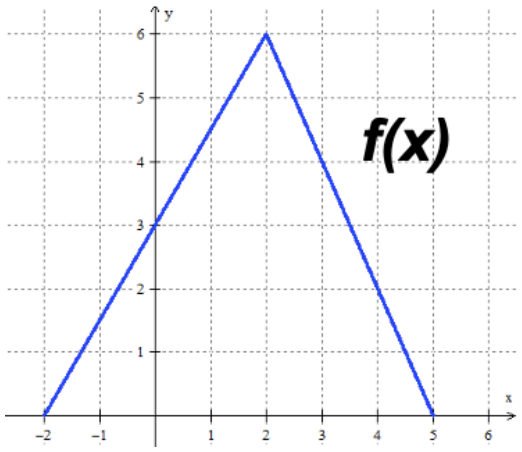
(b) $h'(4)$, where $h(x) = g(f(x))$

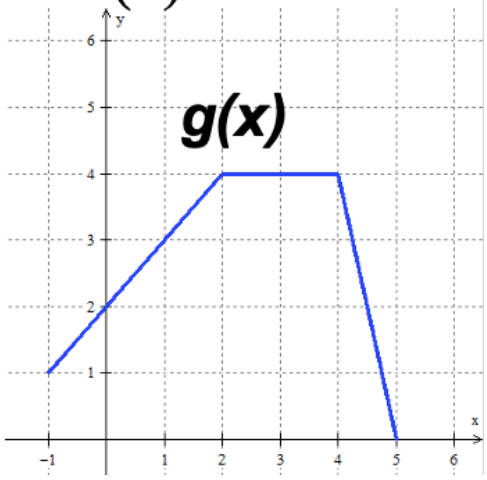
EX. 9: Given the graphs below and

$h(x) = f(g(x))$, $p(x) = g \circ f(x)$

find:

$h'(1)$, $p'(3)$, *and* $h'(2)$





Ex. 10: If $f(x) = \sqrt{5x}$, then $f'(20) =$