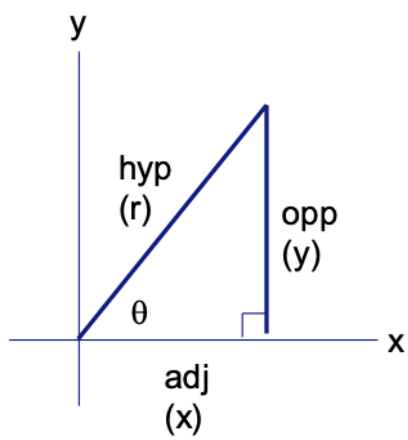
Lesson: Derivative Techniques 2

❖ Obj - Derivatives of Trig Functions

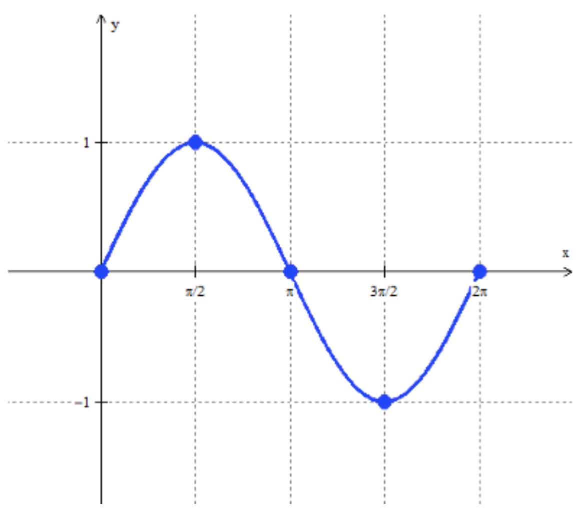
Trig Essentials SOHCAHTOA

$\sin\theta = \frac{y}{r}$	$csc\theta = \frac{r}{y}$
$\cos\theta = \frac{x}{r}$	$sec\theta = \frac{r}{x}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$



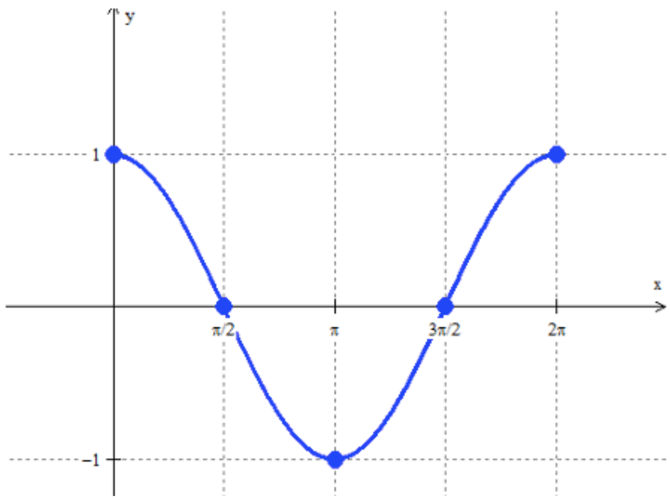
$$y = \sin(x)$$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin x$	0	1	0	-1	0



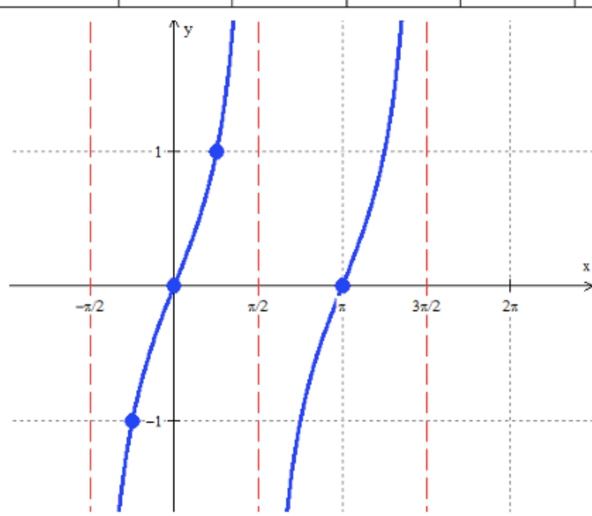
$y = \cos(x)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \cos x$	1	0	-1	0	1



$y = \tan(x)$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$y = \tan x$	$\varnothing$	-1	0	1	$\varnothing$	0	$\varnothing$

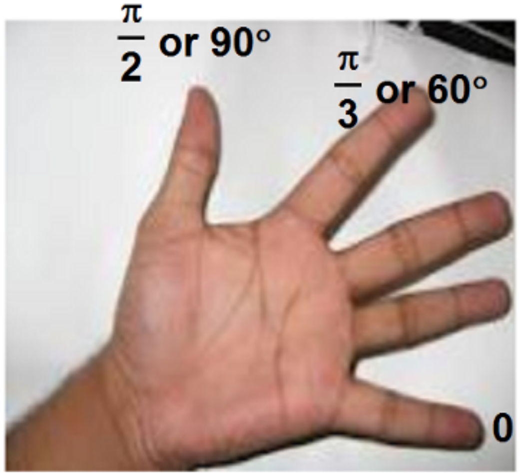


Gang Signs

$$\cos = \frac{\sqrt{\text{fingers above}}}{2}$$

$$\sin = \frac{\sqrt{\text{fingers below}}}{2}$$

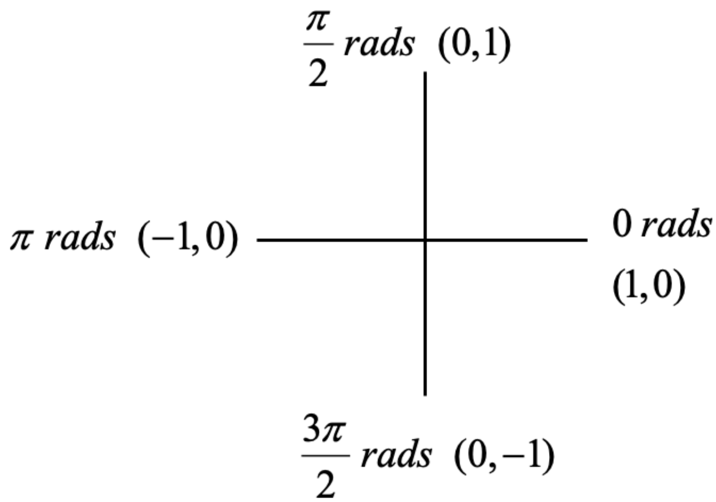
$$\tan = \frac{\sin}{\cos}$$



Practice!

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$					
$\cos x$					
$\tan x$					

Other trig values



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$					
$\cos x$					
$\tan x$					

Basic Trig Identities

$$\frac{1}{\sin x} = \csc x \qquad \frac{\sin x}{\cos x} = \tan x$$

$$\frac{1}{\cos x} = \sec x \qquad \frac{\cos x}{\sin x} = \cot x$$

$$\frac{1}{\tan x} = \cot x \qquad \sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x \qquad \cos^2 x = 1 - \sin^2 x$$

$$\sin(2x) = 2 \cdot \sin x \cdot \cos x$$

Trig Derivative Equations:

1. $$\frac{d}{dx}[\sin x] = \cos x$$

2. $$\frac{d}{dx}[\cos x] = -\sin x$$

Proof that $\frac{d}{dx}[\sin x] = \cos x$

First, a couple of common limits you should know:

$$\lim_{h \to 0} \frac{\sin(h)}{h} = 1$$

$$\lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0$$

And who could forget our friend, the difference quotient:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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- What about Tangent, Cotangent, Secant, and Cosecant?

Let's derive them!

$$\frac{d}{dx}[Tanx] = \frac{d}{dx}\left[\frac{Sinx}{Cosx}\right]$$

- Use quotient rule:

$$\left(\frac{f}{g}\right)' = \frac{g\cdot f' - f\cdot g'}{g^2}$$

- So,

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

EX. 1: Find dy/dx if $y = x \cdot Sinx$

EX. 2: Find dy/dx if $y = \frac{Sinx}{1 + Cosx}$

EX.3 : Find $f''\left(\frac{\pi}{4}\right)$ $if \quad f(x) = Secx$